# Homework 2

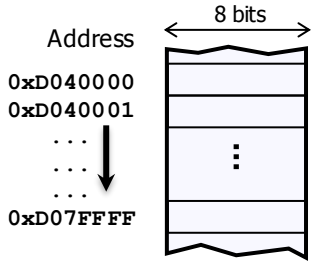
(Due date: October 3rd @ 5:30 pm)

Presentation and clarity are very important! Show your procedure!

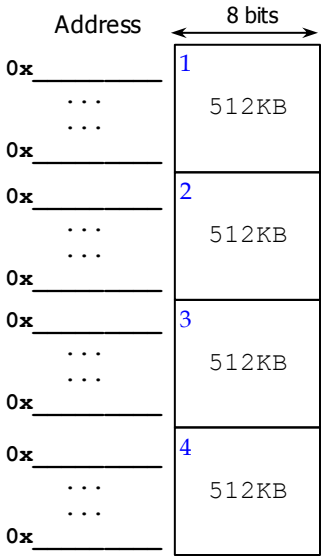
## PROBLEM 1 (28 PTS)

a) What is the minimum number of bits required to represent: (2 pts)
   - ✓ 141,000 symbols?
   - ✓ Numbers between 0 to 16384?

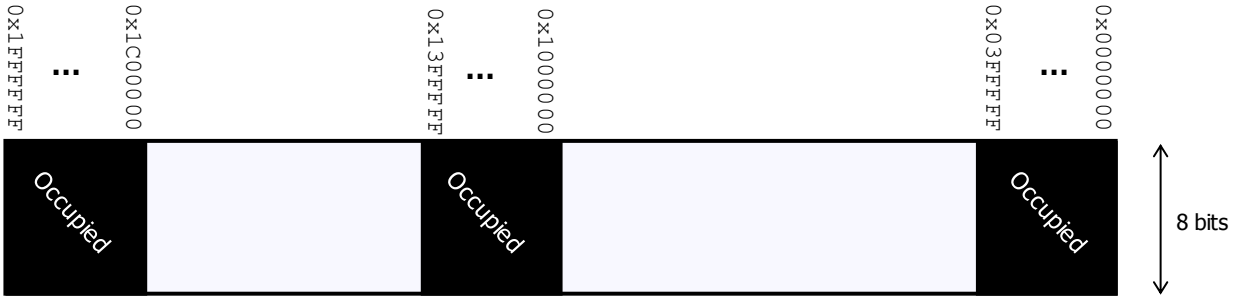
b) A microprocessor has a 28-bit address line. The size of the memory contents of each address is 8 bits. The memory space is defined as the collection of memory positions the processor can address. (6 pts)
   - What is the address range (lowest to highest, in hexadecimal) of the memory space for this microprocessor? What is the size (in bytes, KB, or MB) of the memory space? 1KB = $2^{10}$ bytes, 1MB = $2^{20}$ bytes, 1GB = $2^{30}$ bytes
   - A memory device is connected to the microprocessor. Based on the size of the memory, the microprocessor has assigned the addresses `0xD040000` to `0xD07FFFF` to this memory device.
     - What is the size (in bytes, KB, or MB) of this memory device?
     - What is the minimum number of bits required to represent the addresses only for this memory device?




c) A microprocessor has a memory space of 2 MB. Each memory address occupies one byte. (8 pts)
   - What is the address bus size (number of bits of the address) of this microprocessor?
   - What is the range (lowest to highest, in hexadecimal) of the memory space for this microprocessor?
   - The figure to the right shows four memory chips that are placed in the given positions:
     - Complete the address ranges (lowest to highest, in hexadecimal) for each of the memory chips.




d) The figure below depicts the entire memory space of a microprocessor. Each memory address occupies one byte. (12 pts)
   - What is the size (in bytes, KB, or MB) of the memory space? What is the address bus size of the microprocessor?
   - If we have a memory chip of 4MB, how many bits do we require to address 4MB of memory?
   - We want to connect the 4MB memory chip to the microprocessor. For optimal implementation, we must place those 4MB in an address range where every single address shares some MSBs (e.g.: `0x1C00000` to `0x1FFFFFF`). Provide a list of all the possible address ranges that the 4MB memory chip can occupy. You can only use the non-occupied portions of the memory space as shown below.



# PROBLEM 2 (30 PTS)

- In ALL these problems, you MUST show your conversion procedure. **No procedure ≡ zero points**.

a) Convert the following decimal numbers to their 2's complement representations: binary and hexadecimal. (12 pts)
   ✓ -137.625, 92.3125, -128.6875, -37.65625.

b) Complete the following table. The decimal numbers are unsigned: (6 pts)

| Decimal | BCD | Binary | Reflective Gray Code |
|---|---|---|---|
| 137 | | | |
| | | 10101011 | |
| | | | 1101101010 |
| | | 1011100 | |
| | | | 110001101 |
| | 100101010111 | | |

c) Complete the following table. Use the fewest number of bits in each case: (12 pts)

| REPRESENTATION | | | |
|---|---|---|---|
| **Decimal** | **Sign-and-magnitude** | **1's complement** | **2's complement** |
| -237 | | | |
| | | | 1001000 |
| | | 1011111 | |
| | 110101 | | |
| | | 01010001 | |
| -128 | | | |

# PROBLEM 3 (38 PTS)

a) Perform the following additions and subtractions of the following unsigned integers. Use the fewest number of bits $n$ to represent both operators. Indicate every carry (or borrow) from $c_0$ to $c_n$ (or $b_0$ to $b_n$). For the addition, determine whether there is an overflow. For the subtraction, determine whether we need to keep borrowing from a higher bit. (8 pts)

*Example (n=8):*

✓ 54 + 210

| | $c_8$=1 | $c_7$=1 | $c_6$=1 | $c_5$=1 | $c_4$=0 | $c_3$=1 | $c_2$=1 | $c_1$=0 | $c_0$=0 | |
|---|---|---|---|---|---|---|---|---|---|---|
| 54 = 0x36 = | | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | + |
| 210 = 0xD2 = | | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | |
| Overflow! → | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | |

✓ 77 - 194

Borrow out! →

| | $b_8$=1 | $b_7$=0 | $b_6$=0 | $b_5$=0 | $b_4$=0 | $b_3$=0 | $b_2$=1 | $b_1$=0 | $b_0$=0 | |
|---|---|---|---|---|---|---|---|---|---|---|
| 77 = 0x4D = | | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | - |
| 194 = 0xC2 = | | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | |
| | | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | |

✓ 191 + 201
✓ 210 + 69
✓ 130 - 142
✓ 241 - 36

b) We need to perform the following operations, where numbers are represented in 2's complement: (24 pts)
   ✓ 489 + 23
   ✓ 256 - 87
   ✓ -129 + 126
   ✓ -255 - 231
   ✓ -35 + 66
   ✓ 985 + 122

- For each case:
  - ✓ Determine the minimum number of bits required to represent both summands. You might need to sign-extend one of the summands, since for proper summation, both summands must have the same number of bits.
  - ✓ Perform the binary addition in 2's complement arithmetic. The result must have the same number of bits as the summands.
  - ✓ Determine whether there is overflow by:
    i. Using $c_n, c_{n-1}$ (carries).
    ii. Performing the operation in the decimal system and checking whether the result is within the allowed range for $n$ bits, where $n$ is the minimum number of bits for the summands.
  - ✓ If we want to avoid overflow, what is the minimum number of bits required to represent both the summands and the result?

c) Get the multiplication results of the following numbers that are represented in 2's complement arithmetic with 4 bits. (6 pts)
   ✓ 0101×0101, 1001×0110, 1000×1010.

# PROBLEM 4 (8 PTS)

- The following circuit includes a 4-bit bidirectional port. Complete the timing diagram (signals $DO$ and $DATA$) of the following circuit. The 4-bit binary to gray decoder treats input data as unsigned numbers.